Introduction to FCS

Laboratory for Fluorescence Dynamics
Department of Biomedical Engineering
University of California, Irvine
Outline

What is diffusion?
  Diffusion of molecules
  Diffusion of light
  Generalization of the concept of diffusion
    Translational diffusion
    Rotational diffusion

Some general rules about diffusion
  Diffusion in restricted spaces
  Diffusion in presence of flow
  Anomalous diffusion

How to measure diffusion?
  Macroscopic methods
  Molecular level methods
    Fluctuation based methods
    Single particle tracking methods
Original observation by Brown: the Brownian motion
(From A Fowler lectures)

In 1827 the English botanist Robert Brown noticed that pollen grains suspended in water jiggled about under the lens of the microscope, following a zigzag path. Even more remarkable was the fact that pollen grains that had been stored for a century moved in the same way.

In 1889 G.L. Gouy found that the "Brownian" movement was more rapid for smaller particles (we do not notice Brownian movement of cars, bricks, or people).

In 1900 F.M. Exner undertook the first quantitative studies, measuring how the motion depended on temperature and particle size.

The first good explanation of Brownian movement was advanced by Desaulx in 1877: "In my way of thinking the phenomenon is a result of thermal molecular motion in the liquid environment (of the particles)." This is indeed the case. A suspended particle is constantly and randomly bombarded from all sides by molecules of the liquid. If the particle is very small, the hits it takes from one side will be stronger than the bumps from other side, causing it to jump. These small random jumps are what make up Brownian motion.
In 1905 A. Einstein explained Brownian motion using energy equipartition: the kinetic theory of gases developed by Boltzmann and Gibbs could explain the randomness of the motion of large particles without contradicting the Second Principle of Thermodynamics. This was the first “convincing” proof of the particle nature of matter as declared by the adversaries of atomism.

The role of fluctuations
Explanation in terms of fluctuations

Why measure diffusion with optical methods?
   Diffusion in microscopic spaces
   Diffusion in biological materials

Difference between diffusion, “hopping” and binding
Introduction to Fluorescence Correlation Spectroscopy (FCS)

Fluctuation analysis goes back to the beginning of the century

• Brownian motion (1827). Einstein explanation of Brownian motion (1905)
  Noise in resistors (Johnson, Nyquist), white noise {Johnson, J.B., Phys. Rev. 32 pp. 97 (1928), Nyquist, H., Phys. Rev. 32 pp. 110 (1928)}
• Noise in telephone-telegraph lines. Mathematics of the noise. Spectral analysis
  • I/f noise
  • Noise in lasers (1960)
  • Dynamic light scattering
  • Noise in chemical reactions (Eigen)
• FCS (fluorescence correlation spectroscopy, 1972) Elson, Magde, Webb
• FCS in cells, 2-photon FCS (Berland et al, 1994)
• First commercial instrument (Zeiss 1998)
Why we need FCS to measure the internal dynamics in cell??

Methods based on perturbation
   Typically FRAP (fluorescence recovery after photobleaching)

Methods based on fluctuations
   Typically FCS and dynamic ICS methods

There is a fundamental difference between the two approaches, although they are related as to the physical phenomena they report.
Introduction to “number” fluctuations

In any open volume, the number of molecules or particles fluctuate according to a Poisson statistics (if the particles are not-interacting)

The average number depends on the concentration of the particles and the size of the volume

The variance is equal to the number of particles in the volume

This principle does not tell us anything about the time of the fluctuations
If we perturb a system from **equilibrium**, it returns to the average value with a characteristic time that depends on the process responsible for returning the system to equilibrium.

**Spontaneous** energy fluctuations in a part of the system, can cause the system to locally go out of equilibrium. These spontaneous fluctuations **dissipate** with the same time constant as if we had externally perturbed the equilibrium of the system.
First Application of Correlation Spectroscopy
(Svedberg & Inouye, 1911) Occupancy Fluctuation

Experimental data on colloidal gold particles:

| 120002001324123102111131125111102331333221122422122612214 |
| 2345241141311423100100421123123201111000111_211001320000 |
| 10011000100023221002110000201001_333122000231221024011102_ |
| 1222112231000110331110210110010103011312121010121111211_10 |
| 00322101230201212321110110023312242110001203010100221734 |
| 41010100211221144442121144013212331431301122212331012111 |
| 222412231113322132110000410432012120011322231200_253212033 |
| 23311110021002201301132111312001013143221122112323442230 |
| 321421532200202142123232043112312003314223452134110412322 |
| 220221 |

Collected data by counting (by visual inspection) the number of particles in the observation volume as a function of time using a “ultra microscope”
Particle Correlation and spectrum of the fluctuations

*Histogram of particle counts
*Poisson behavior
*Autocorrelation not available in the original paper. It can be easily calculated today.

Comments to this paper conclude that scattering will not be suitable to observe single molecules, but fluorescence could
What can cause a fluctuation in the fluorescence signal???

- Number of fluorescent molecules in the volume of observation, diffusion or binding
- Conformational Dynamics
- Rotational Motion if polarizers are used either in emission or excitation
- Protein Folding
- Blinking
- And many more

Example of processes that could generate fluctuations

Each of the above processes has its own dynamics. FCS can recover that dynamics
Generating Fluctuations By Motion

What is Observed?

1. The rate of motion
2. The concentration of particles
3. Changes in the particle fluorescence while under observation, for example conformational transitions

Note that the spectrum of the fluctuations responsible for the motion is “flat” but the spectrum of the fluorescence fluctuations is not.
Defining Our Observation Volume under the microscope: One- & Two-Photon Excitation.

1 - Photon

 Defined by the pinhole size, wavelength, magnification and numerical aperture of the objective

Approximately 1 \( \text{um}^3 \)

2 - Photon

 Defined by the wavelength and numerical aperture of the objective
Why confocal detection?

*Molecules are small, why to observe a large volume?*

- Enhance signal to background ratio
- Define a well-defined and reproducible volume

Methods to produce a confocal or small volume

( limited by the wavelength of light to about 0.1 fL)

- Confocal pinhole
- Multiphoton effects
  - 2-photon excitation (TPE)
  - Second-harmonic generation (SGH)
  - Stimulated emission
  - Four-way mixing (CARS)

(not limited by light, not applicable to cells)

- Nanofabrication
- Local field enhancement
- Near-field effects
Need a pinhole to define a small volume.
\[ n_a \approx \frac{d}{\tau} \left( \frac{p \pi A^2}{fhc\lambda} \right)^2 \]

- \( n_a \): Photon pairs absorbed per laser pulse
- \( p \): Average power
- \( \tau \): Pulse duration
- \( f \): Laser repetition frequency
- \( A \): Numerical aperture
- \( \lambda \): Laser wavelength
- \( d \): Cross-section

From Webb, Denk and Strickler, 1990
Laser technology needed for two-photon excitation

Ti:Sapphire lasers have pulse duration of about 100 fs

Average power is about 1 W at 80 MHz repetition rate

About 12.5 nJ per pulse (about 125 kW peak-power)

Two-photon cross sections are typically about

$$\delta = 10^{-50} \text{ cm}^4 \text{ sec photon}^{-1} \text{ molecule}^{-1}$$

Enough power to saturate absorption in a diffraction limited spot
### Orders of magnitude (for 1 μM solution, small molecule, water)

<table>
<thead>
<tr>
<th>Volume</th>
<th>Device</th>
<th>Size(μm)</th>
<th>Molecules</th>
<th>Time</th>
</tr>
</thead>
<tbody>
<tr>
<td>milliliter</td>
<td>cuvette</td>
<td>10000</td>
<td>6x10^{14}</td>
<td>10^4</td>
</tr>
<tr>
<td>microliter</td>
<td>plate well</td>
<td>1000</td>
<td>6x10^{11}</td>
<td>10^2</td>
</tr>
<tr>
<td>nanoliter</td>
<td>microfabrication</td>
<td>100</td>
<td>6x10^8</td>
<td>1</td>
</tr>
<tr>
<td>picoliter</td>
<td>typical cell</td>
<td>10</td>
<td>6x10^5</td>
<td>10^{-2}</td>
</tr>
<tr>
<td>femtoliter</td>
<td>confocal volume</td>
<td>1</td>
<td>6x10^2</td>
<td>10^{-4}</td>
</tr>
<tr>
<td>attoliter</td>
<td>nanofabrication</td>
<td>0.1</td>
<td>6x10^{-1}</td>
<td>10^{-6}</td>
</tr>
</tbody>
</table>
Determination of diffusion by fluctuation spectroscopy
The time series

The autocorrelation function

N and relaxation time of the fluctuation

The histogram of the counts in a given time bin (PCH). 
N and brightness

Duration

Detail of one time region
How to extract the information about the fluctuations and their characteristic time?

Distribution of the **amplitude** of the fluctuations

Distribution of the **duration** of the fluctuations

To extract the distribution of the duration of the fluctuations we use a math based on calculation of the **correlation function**

To extract the distribution of the amplitude of the fluctuations, we use a math based on the **PCH distribution**
The definition of the Autocorrelation Function

\[ \delta F(t) = F(t) - \langle F(t) \rangle \]

\[ G(\tau) = \frac{\langle \delta F(t) \delta F(t + \tau) \rangle}{\langle F(t) \rangle^2} \]
What determines the intensity of the fluorescence signal??

This is the fundamental equation in FCS

\[ F(t) = \kappa Q \int d\mathbf{r} \, W(\mathbf{r})C(\mathbf{r}, t) \]

- \( \kappa Q \): quantum yield and detector sensitivity (how bright is our probe). This term could contain the fluctuation of the fluorescence intensity due to internal processes.
- \( W(\mathbf{r}) \): describes the profile of illumination.
- \( C(\mathbf{r}, t) \): is a function of the fluorophore concentration over time. This is the term that contains the “physics” of the diffusion processes.

The value of \( F(t) \) depends on the profile of illumination!
What about the excitation (or observation) volume shape?

For the 2-photon case, these expression must be squared

More on the PSF in Jay’s lecture

$$F(x, y, z) = I_0 I(z) e^{-2(x^2+y^2)/w_0^2}$$

$$I(z) = \exp\left(-\frac{2z^2}{w_{0z}}\right) \quad \text{Gaussian } z$$

$$I(z) = \frac{1}{1+\left(\frac{z}{w_{0z}}\right)^2} \quad \text{Lorentzian } z$$
The Autocorrelation Function

In the simplest case, two parameters define the autocorrelation function: the amplitude of the fluctuation ($G(0)$) and the characteristic relaxation time of the fluctuation.

$G(0) \propto 1/N$

As time (tau) approaches 0
The Effects of Particle Concentration on the Autocorrelation Curve

Observation volume

\[ <N> = 2 \]

\[ <N> = 4 \]
A Poisson distribution describes the statistics of particle occupancy fluctuations. For a Poisson distribution the variance is proportional to the average:

\[ \langle N \rangle = \langle \text{Particle \_ Number} \rangle = \text{Variance} \]
G(0), Particle Brightness and Poisson Statistics

Average = 0.275

\[ \langle N \rangle \propto \frac{\text{Average}^2}{\text{Variance}} = \frac{0.275^2}{0.256} = 0.296 \]

Variance = 0.256

Let's increase the particle brightness by 4x:

Average = 1.1

\[ \langle N \rangle \propto 0.296 \]

Variance = 4.09
For a 2-dimensional Gaussian excitation volume:

\[ G(\tau) = \frac{\gamma}{N} \left( 1 + \frac{4D\tau}{w_{2DG}^2} \right)^{-1} \]

2-photon equation contains a 8, instead of 4

For a 3-dimensional Gaussian excitation volume:

\[ G(\tau) = \frac{\gamma}{N} \left( 1 + \frac{4D\tau}{w_{3DG}^2} \right)^{-1} \left( 1 + \frac{4D\tau}{z_{3DG}^2} \right)^{-1/2} \]

3D Gaussian “time” analysis: with \( \tau_D = w^2/4D \) and \( S = w/z \)

\[ G(\tau) = \frac{\gamma}{N} \left( 1 + \frac{\tau}{\tau_D} \right)^{-1} \left( 1 + S^2 \cdot \frac{\tau}{\tau_D} \right)^{-1/2} \]
Blinking and binding processes

Until now, we assumed that the particle is not moving. If we assume that the blinking of the particle is independent on its movement, we can use a general principle that states that the correlation function splits in the product of the two independent processes.

\[
G_{Total}(\tau) = G_{Blinking}(\tau) \cdot G_{Diffusion}(\tau)
\]

\[
G_{Binding}(\tau) = \left[ 1 + K\left( f_A - \frac{f_B}{K} \right)^2 e^{-\lambda \tau} \right]
\]

\(K = k_f / k_b\) is the equilibrium coefficient; \(\lambda = k_f + k_b\) is the apparent reaction rate coefficient; and \(f_j\) is the fractional intensity contribution of species \(j\)
How different is $G(\text{binding})$ from $G(\text{diffusion})$?

With good S/N it is possible to distinguish between the two processes. Most of the time diffusion and exponential processes are combined.
The Effects of Particle Size on the Autocorrelation Curve

Diffusion Constants

- 300 um²/s
- 90 um²/s
- 71 um²/s

Stokes-Einstein Equation:

\[ D = \frac{k \cdot T}{6 \cdot \pi \cdot \eta \cdot r} \]

and

\[ MW \propto Volume \propto r^3 \]

Monomer --> Dimer

Only a change in D by a factor of \(2^{1/3}\), or 1.26
Autocorrelation Adenylate Kinase -EGFP Chimeric Protein in HeLa Cells

Examples of different *Hela* cells transfected with AK1-EGFP

Examples of different *Hela* cells transfected with AK1β -EGFP

Qiao Qiao Ruan, Y. Chen, M. Glaser & W. Mantulin Dept. Biochem & Dept Physics- LFD Univ II, USA
Normalized autocorrelation curve of EGFP in solution (•), EGFP in the cell (●), AK1-EGFP in the cell (●), AK1β-EGFP in the cytoplasm of the cell (○).
Autocorrelation of Adenylate Kinase –EGFP on the Membrane

Clearly more than one diffusion time

A mixture of AK1b-EGFP in the cytoplasm and membrane of the cell.
Diffusion constants (um²/s) of AK EGFP-AKβ in the cytosol - EGFP in the cell (HeLa). At the membrane, a dual diffusion rate is calculated from FCS data. Away from the plasma membrane, single diffusion constants are found.
Two Channel Detection: Cross-correlation

1. Increases signal to noise by isolating correlated signals.
2. Corrects for PMT noise

Sample Excitation Volume

Beam Splitter

Detector 1

Detector 2

Each detector observes the same particles
Removal of Detector Noise by Cross-correlation

11.5 nM Fluorescein

Detector after-pulsing

Detector 1

Detector 2

Cross-correlation
Calculating the Cross-correlation Function

Detector 1: $F_i$

$\tau$

$t$

$t + t$

Detector 2: $F_j$

$$G_{ij}(\tau) = \frac{\langle dF_i(t) \cdot dF_j(t + \tau) \rangle}{\langle F_i(t) \rangle \cdot \langle F_j(t) \rangle}$$
Cross-correlation calculations

One uses the same fitting functions you would use for the standard autocorrelation curves.

Thus, for a 3-dimensional Gaussian excitation volume one uses:

\[
G_{12}(\tau) = \frac{\gamma}{N_{12}} \left( 1 + \frac{4D_{12} \tau}{w^2} \right)^{-1} \left( 1 + \frac{4D_{12} \tau}{z^2} \right)^{-1/2}
\]

\(G_{12}\) is commonly used to denote the cross-correlation and \(G_1\) and \(G_2\) for the autocorrelation of the individual detectors. Sometimes you will see \(G_x(0)\) or \(C(0)\) used for the cross-correlation.
Two-Channel Cross-correlation

The cross-correlation ONLY if particles are observed in both channels.

Each detector observes particles with a particular color.

The cross-correlation signal:

Only the green-red molecules are observed!!
Two-color Cross-correlation

Equations are similar to those for the cross correlation using a simple beam splitter:

\[ G_{ij}(\tau) = \frac{\langle dF_i(t) \cdot dF_j(t + \tau) \rangle}{\langle F_i(t) \rangle \cdot \langle F_j(t) \rangle} \]

### Information Content

<table>
<thead>
<tr>
<th>Correlated signal from particles having both colors.</th>
<th>( G_{12}(\tau) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Autocorrelation from channel 1 on the green particles.</td>
<td>( G_1(\tau) )</td>
</tr>
<tr>
<td>Autocorrelation from channel 2 on the red particles.</td>
<td>( G_2(\tau) )</td>
</tr>
</tbody>
</table>
Experimental Concerns: Excitation Focusing & Emission Collection

assume exact match of the observation volumes in our calculations which is difficult to obtain experimentally.

**Excitation side:**
1. Laser alignment
2. Chromatic aberration
3. Spherical aberration

**Emission side:**
1. Chromatic aberrations
2. Spherical aberrations
3. Improper alignment of detectors or pinhole (cropping of the beam and focal point position)
For two uncorrelated species, the amplitude of the cross-correlation is proportional to:

\[
G_{12}(0) \propto \left[ \frac{f_{11}f_{12}\langle N_1 \rangle + f_{21}f_{22}\langle N_2 \rangle}{f_{11}f_{12}\langle N_1 \rangle^2 + (f_{11}f_{22} + f_{21}f_{12})\langle N_1 \rangle\langle N_2 \rangle + f_{21}f_{22}\langle N_2 \rangle^2} \right]
\]
Simulation of cross-correlation
Same diffusion $= 10 \, \mu \text{m}^2/\text{s}$

<table>
<thead>
<tr>
<th></th>
<th>Channel 1</th>
<th>Channel 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Molecules</td>
<td>100</td>
<td></td>
</tr>
<tr>
<td>Molecules</td>
<td>50</td>
<td></td>
</tr>
<tr>
<td>Molecules</td>
<td>20</td>
<td>20</td>
</tr>
</tbody>
</table>

Correlation plot (log averaged)

The fraction cannot be larger than the smallest of the $G(0)$'s
Discussion

1. The PSF: how much it affects our estimation of the processes?
2. Models for diffusion, anomalous?
3. Binding?
4. FRET (dynamic FRET)?
5. Bleaching?

6. ......and many more questions